LETTERS TO THE EDITOR

To the Editor:

1. In a paper entitled "Critical Impeller Speed for Solid Suspension in Mechanically Agitated Contactors," Rao et al. [**34**(8), Aug. 1988, p. 1322] state that a study of solids suspension using pitched-blade turbines was undertaken because these impellers have received only scant attention. Actually, agitated suspensions using pitched-blade turbines have been researched extensively in the last ten years, with studies by Musil (1976), Musil and Vlk (1978), Rieger et al. (1978), Herringe (1979), Musil (1984), Musil et al. (1984), Buurman et al. (1985), Dilt (1985), Dilt and Rieger (1985), and Chudacek (1986). The work by Chudacek (1986) is the most comprehensive to date. Gates et al. (1976) presented a design procedure for suspensions with pitched-blade turbines based on experimental data which was not published. In addition to the review on agitated solids suspensions in Uhl and Gray (1986) mentioned by Rao et al., A. W. Nienow has presented in-depth studies in Harnby et al. (1985) and Ulbrecht and Patterson (1985).

2. Rao et al. (1988, Figure 2) show that the power number gradually increases as particles are suspended and then levels off when the solids suspension is complete, the point, N, of leveling off corresponding to the point of complete suspension, N_{CS}. Herringe (1979) considered this matter in detail and showed that, at the point of complete suspension, the power number actually goes through a maximum.

3. Rao et al. cite the type of correlation developed by Zwietering (1958) as being particularly useful:

$$N_{CS} = s\gamma^{0.1}d_p^{0.2} \cdot (g\Delta\rho/\rho_L)^{0.45}X^{0.13}/D^{0.85} \quad (1)$$

(Zwietering actually used $[100\ X/(100 - X)]$ for the concentration term). Rao et al. used Zwietering's term of $\gamma^{0.1}$ in their equation, even though they did not investigate the effect of the viscosity of the suspending medium on N_{CS}. Zwietering initially determined that the exponents on d_p and the concentration, X, were constant at 0.20 and 0.13, respectively. However, the value of x on D^x (Eq. 1), the scale factor, varied from -0.78 to -0.94, a fact not generally recognized. Due to this range of x values, Zwietering stated that his dimensional analysis necessitated that the value of the exponent on γ, lie between 0.05 and 0.15, while the exponent on g (and $\Delta\rho/\rho_L$) should range from 0.42 to 0.47. The effect of viscosity and solid density were not determined independently. It was determined simultaneously that N_{CS} varied with $\gamma^{0.1}$ and $(g\Delta\rho/\rho_L)^{0.45}$, and because of these exponents, x had to equal -0.85. Therefore, the values of these three exponents were forced to make the overall equation dimensionless, and no data were presented to indicate that the effect of viscosity on N_{CS} was indeed positive.

The only worker investigating and reporting the effect of viscosity independently, found the reverse. Einenkel (1980) made some determinations with a propeller on glass beads in corn syrup solutions with the viscosity varying 96 fold. The data have been replotted as N vs. viscosity, with suspension criteria as a parameter (Figure 1 of this letter). Straight lines with slopes of -0.16 indicate that N varies with $\gamma^{-0.16}$. Pavlushenko et al. (1957) reported that for unbaffled vessels, the effect of viscosity on the speed to suspend solids, as determined empirically, varied with $\gamma^{-0.20}$. Other workers have reported positive values of the exponent, but almost all of these result from correlations using the Reynolds number, and the actual effect of the viscosity was not indicated. Illustrative of this is the correlation of Einenkel (1980) who plotted a theoretical expression vs. the Reynolds number, which indicated that N_{CS} should vary with $\gamma^{0.08}$, in contradiction to the actual viscosity data he presented (Figure 1, this letter). There is no evidence that Zwietering's viscosity term is an approximation in the correct direction.

4. Rao et al. found that N_{CS} varied with $d_p^{0.11}$ for particles ranging from 100 to 2,000 μm. Zwietering had originally found that the exponent was 0.20 for 130 to 800 μm particles. A number of later workers obtained values of from 0.15 to 0.26. On the other hand, Kneule and Weinspach (1967) found the exponent to vary from 0 to -0.2, while Rieger et al. (1978) reported a value of -0.1. All of these workers felt that the value of the exponent was constant over the range of particle sizes studied.

Two researchers have published N_{CS} vs. d_p data which show that the exponent on d_p is not constant, but changes with particle size. The largest range of particle sizes was run by Herringe (1979): sand particles from 19 to 5,000 μm, a range of 263:1. The data for a 300-mm vessel with a six-blade, 45° turbine are shown in Figure 2 of this letter. For the smallest particle sizes, the slope is about 0.25. It increases to a maximum of 0.54 between 125 and 300 μm, and then falls off slowly until the slope is zero for the two largest particle sizes. Herringe's data is corroborated by Einenkel (1980) for 88 to 1,430 μm mullite particles ($\rho_S = 3{,}000$ kg/m^3) in water with a propeller; the shape of the curve is almost identical to that of Herringe's (Figure 2, this letter). Chudacek (1986) obtained a value of 0.52 for sand particles ranging from 77 to 290 μm (Figure 2). The single particle size (157 μm)

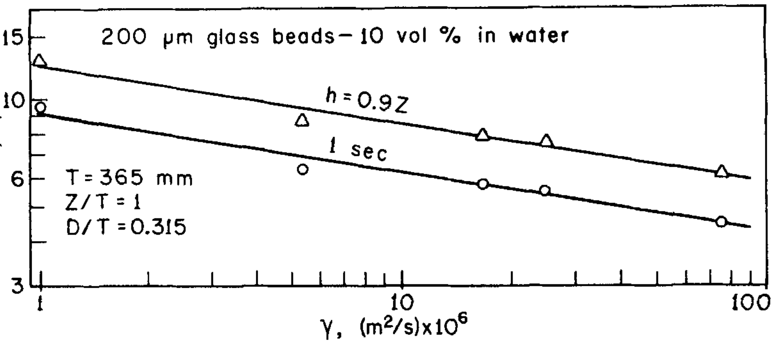

Figure 1. Effect of suspending medium's viscosity on critical impeller speed for 1 sec and 90% slurry height criteria.

used by Buurman et al. (1985) extrapolated to $T = 300$ mm is also shown (Figure 2). Herringe's curve explains the exponents obtained by Rieger et al. (−0.1) and Kneule and Weinspach (0 to −0.2); they used very large particles and the curve may possibly have a negative slope at these large particle sizes.

Data taken from Rao et al. (Figure 4a) for 23.6 wt. % (11 vol. %) in a 570 mm vessel corrected to 300 mm, are shown in Figure 2 of this letter. The points for Rao et al.'s 700 and 2,000 μm particles fall slightly above Herringe's curve, the 340 μm point is 20% above, while the 100 μm point is 100% above Herringe's curve. All of Rao et al.'s data points should actually be considerably below Herringe's, as they used impellers with $W/D = 0.3$, which had 50% more area than the standard $W/D = 0.2$, which Herringe used. The reasons for the generally high N_{CS} values and the low exponent on d_p for the 100 μm particles, are not apparent.

5. Rao et al. (Figure 8, 1988) determined that N_{CS} varied with $(D/T)^{-1.16}$, and noted that the exponent was lower than the −1.67 obtained by Zwietering (1958) or the −1.5 found by Chapman et al. (1983). Not mentioned is Kolar's (1961) value of −1.54 for axial flow impellers using as a criterion, equal solids compositions at heights of $T/4$ below the surface and off the bottom, or the value of −2.33 used in Gates et al.'s equation (1976). The reason for the rather great difference between Rao et al.'s value and all other published data is not evident.

6. Rao et al. determined that N_{CS} varied with scale as did $T^{-0.85}$ (based on 100 μm particles), and noted that this compared favorably with Zwietering's −0.85 and Chapman et al.'s −0.76 (1983). There are a number of other values for 45° pitched blade turbines which could be cited: Gates et al. (1976) used a value of −0.75, Buurman et al. (1985) determined a value of −0.67 for a particle size of 157 μm, while most recently Chudacek (1986) found a value of −0.58 for 77 to 290 μm particles. But most importantly, Herringe (1979) found that the scale exponent varied with particle size, decreasing from about −0.85 for the smallest particles (20 to 200 μm) to about −0.4 for the largest (1,000 to 5,000 μm). This is corroborated by Kneule and Weinspach (1967) who found a value of −0.5 for very large particles. Rao et al.'s value of −0.85 may be valid for small particles but it is questionable that it can be extended to 2,000 μm particles as they suggest.

7. Rao et al. state that they used "standard" baffles of 10% width. In the United States, standard baffle width in industry is universally 8.3% ($T/12$) (Bates et al., 1963; Uhl and Gray, 1966; Oldshue, 1983). The unit for d_p in equations is m, not μm as shown in the Notation.

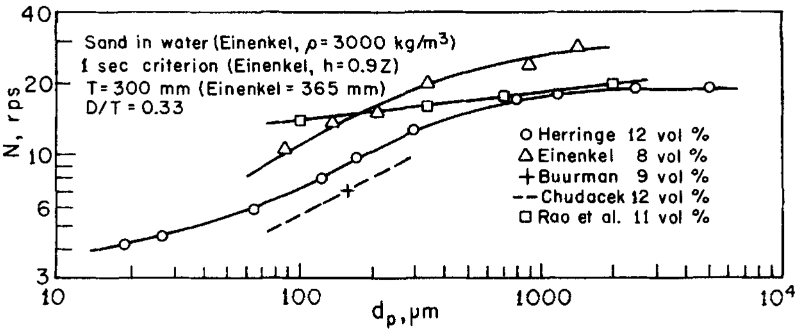

Figure 2. Effect of particle size on critical impeller speed for various axial flow impellers.

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Richard L. Bowen, Jr.
Tensco
Barrington, RI

Reply:

We appreciate the interest shown by Bowen in our paper. Following are the pointwise answers:

1. The studies on solid suspensions in mechanically agitated reactors are summarized in Table 1. Out of the nineteen studies, the majority of experimentation was carried out in small diameter vessels (0.1–0.5 m) except those by Herringe (1979), Einenkel (1980), Buurman et al. (1985), and Chudacek (1986). However, these later investigators selected only one ratio of impeller diameter to vessel diameter (D/T) and carried out only a few experiments. In all these cases, the pitched blade downflow turbine (*PTD*) had six blades. Chapman et al. (1983) employed two four-bladed impellers in the 0.56 m I.D. vessel. Further, the effects of blade width and blade thickness have not been systematically investigated in the past except a few experiments by Buurman (1985). For the development of generalized and rational correlation it is desirable to have a complete set of data covering a wide rage of variables. Raghav Rao et al. (1988) have presented such a set of data. They employed 0.3, 0.57, 1.0, and 1.5 m I.D. vessels and the D/T ratio was varied in each. The ratio of blade width to impeller diameter (W/D) was varied in the range of 0.25 to 0.40. The effect of blade thickness was also investigated. They also covered impeller clearance and particle settling velocity over a wide range. Based on these data, Raghav Rao et al. (1989) and Rewatkar et al. (1989) have now developed rational correlations.

The incomplete nature of the previous work has also been systematically pointed out by Bowen (1989). He has clearly brought out the discrepancies in the published literature as regards the effects of liquid viscosity and particle size, impeller diameter, and vessel diameter.

2. A typical variation of power number (N_p) with impeller speed, N, based on our observations, is shown in Figure 1. As stated earlier (Raghav Rao et al., 1988), we have not observed any maximum in N_p-N relationship in 0.57, 1.0, and 1.5 m I.D. vessels, and no decrease in N_p was observed (in the range of particle settling velocity covered) after complete suspension of solids. We have recently repeated these experiments for all the impellers in all the vessels, at various solid loadings. Herringe (1979) has found a decrease in N_p after complete suspension of solids. This is probably due to surface aeration which is likely to occur in small diameter vessels. We did a systematic set of experiments in the presence of gas. We observed that the N_p-N curve shows a maximum in the presence of gas [Raghav Rao et al. (1989)].

3. While developing a correlation for critical impeller speed for solid suspension, we kept the same form of correlation as that reported by Zwietering (1958). We are of the opinion that the effect of liquid viscosity should be correlated through its role in particle settling velocity. In order to include the effect of liquid viscosity, the solid suspension was studied in the viscosity range of 0.8 to 7.0 MPa · s. As a result of this viscosity variation, the terminal settling velocity of 2,200 μm quartz particles varied in the range of 220 to 113 mm/s. Alternatively the particle settling velocity was varied by changing the particle size. The variation of N_{cs} with particle settling velocity is shown in Figure 2. It was found that the value of N_{cs} varies as $V_{s\infty}^{0.28}$. If this observation is included in the correlation developed by Raghav Rao et al., (1988) the following modified correlation is obtained for 0.3, 0.57, and 1.0 m I.D. vessels:

$$N_{cs} = C_1 \frac{V_{s\infty}^{0.28} X^{.1}}{D^{0.85}} \left(\frac{T}{D}\right)^{0.31} \quad S.D. = 8\% \qquad (1)$$

where the value of C_1 was found to be 1.93. In order to compare our results with those of Herringe (1979), his results are also plotted in Figure 2. It can be seen that, for the range of particle settling velocities covered in this work, the results of both investigations are comparable.

Bowen (1989) observed some difference between our results and those of Herringe (1979) because he based the comparison on the particle size. We recommend that the comparison be made at the same settling velocity (Settling velocities of particles were reported by Raghav Rao, 1988). Two additional investigations support our observation of dependence of N_{cs} on particle settling velocity. The data reported by Chudacek (1986) can be correlated on the basis of settling velocity. It can be shown that the value of N_{cs} varies as $V_{s\infty}^{0.3}$. Gates et al. (1976) found N_{cs} to vary as $V_{s\infty}^{0.267}$. It can be seen that these reported exponents are close to that in Eq. 1. While developing correlation 1, we covered the particle size range of 100 μm to 2,200 μm. It may be a good idea to cover additional particle sizes on both sides and undertake a systematic investigation using large size vessels.

Bowen (1989) has correctly pointed out the limitation of original correlation of Zwietering (1958) as regards the effect of liquid viscosity on N_{cs}. Though we have shown that the effect of viscosity can be included through its effect on particle settling velocity, the same is not true for the density difference. Herringe (1979) has shown this very clearly. He found N_{cs} to vary as $\Delta_\rho^{0.44}$. Nienow (1968) has also systematically studied the effect of $\Delta\rho$ and he found the exponent value, 0.43. If the particle Reynolds number falls in the turbulent regime (>1,000), $V_{s\infty}$ varies as $\Delta\rho^{0.5}$. In this case, Eq. 1 reasonably explains the effect of $\Delta\rho$. However, at lower Reynolds number, the effect of $\Delta\rho$ remains, in addition to its effect through $V_{s\infty}$. Though the reported data are in line with this statement, we feel that this aspect needs to be further investigated. For this purpose, the particle size needs to be varied over a wide range for each density.

4. The N_{cs} values of Raghav Rao et al. (1988) are slighty lower than those of Herringe (1979). Bowen (1989) has pointed out that the N_{cs} values of Raghav Rao et al. (1988) should have been lower because they used blades of higher width ($W/D = 0.3$). It may be emphasized that the criterion for suspension was different. We used the criterion suggested by Zwietering (1958), which states that, "the suspension is considered to be complete when no solid particles remained on the bottom for more than 2 s." On the other hand,

Table 1. Literature Survey on Suspension of Solid-Particles in Mechanically Agitated Contactors (Solid-Liquid System)

Author	Impeller Type	T m	D m	W/D	$k \times 10^3$ m	$\Delta\rho$ kg/m^3	d_p μm	X (w/w, %)	$\mu_L \times 10^3$ kg · m^{-1} · s^{-1}
Zwietering (1958)	2, Paddles 6, *DT* Propellers Vaned Disc	0.154, 0.192, 0.24, 0.29, 0.45, 0.6	0.06 0.08, 0.112, 0.16, 0.224	0.5, 0.25 0.2 0.1	—	510–1,810	125–850	0.5–20	0.32–9.3
Narayanan et al. (1969)	8, Paddle	0.114, 0.141	0.036, 0.046, 0.057	—	—	140–1,600	106–600	2.5–20	1.0
Weinsmann & Efferding (1960)	6, Paddle	0.14, 0.238, 0.289	0.051, 0.1	—	—	1,600–8,700	45–140	10–50	1.0–2.0
Kolar (1961)	Propeller 6, *PTD*	0.165, 0.218 0.345	0.052, 0.07 0.11	—	—	150	570–1,640	3–20	1.0
Nienow (1968)	6, *DT*	0.14	0.0364, 0.049, 0.073	0.2	—	530–1,660	153–9,000	0.1–1.0	1.0
Bourne & Sharma (1974)	Propeller	0.172	0.08	—	—	1,640	220–1,120	0–6.12	1.0
Baldi et al. (1978)	8, *DT*	0.122, 0.19, 0.229	0.032, 0.04, 0.048	0.25	—	1,414–1,800	50–545	0.2–2	0.0645–3.17
Subbarao & Taneja (1979)	3, Propeller	0.164	0.051	—	—	1,666–2,010	63–3,070	30	1.0
Herringe (1979)	6, *PTD*	0.3, 1.0	0.05, 0.1, 0.333	—	—	490–3,470	19–5,000	14.5–45.6	1.0, 10
Bohnet & Niesmark (1980)	Propeller	0.29	0.1	—	—	50–1,480	1,050 2,480	0.7–5.4	1.0
Einenkel (1980)	Propeller	0.365, 0.79	0.12, 0.24	—	—	1,870, 1,460	200–630	12.55–41.8	1.0
Chapman et al. (1983)	6, *DT* 4, *PTU* 4,6 *PTD*	0.29, 0.3, 0.56, 0.91, 1.83	0.14, 0.28 0.457, 0.9 0.79	0.2, 0.3	3.2, 9.5	50–1,900	80–2,800	0–3.0	1.0
Musil et al. (1984)	3, *PTD* 24°, 33°, 45° 6, *PTD*	0.6	0.15, 0.195, 0.245, 0.315	—	—	154	700		1.0, 5
Buurman et al. (1985)	4, *PTD*	0.48, 4.26	0.2, 1.72	0.25	0.5 4.0, 15.0	1,650	157–2,200	7.6 32	1.0
Ditl & Rieger (1985)	3,6 *PTD* 6, *FBT*	0.15, 0.2, 0.3, 0.4	0.10–0.54	—	—	259, 1,600	85–4,000	2.56–12	1–0.47
Chudacek (1985)	3, Propeller 6,4, *ADT*	0.5	0.165	0.2	2.0	1,650	110 290	6.1–24.4	1.0
Chudacek (1986)	3, Propeller 6 *PTD*	0.5, 1.0	0.167, 0.33	0.2	2.9 4.0	1,650	77–290	14.7–46.0	1.0
Gray (1987)	6, *FBT* 3, *PTD* *DT*	0.292	0.064, 0.976, 0.102	—	—	350–2,200	1,000, 3,200	1–10	1.0
Raghav Rao et al. (1988)	6, *DT* 6, *PTU* 6, *PTD*	0.3, 0.4, 0.57, 1.0 1.5	0.19, 0.285 0.19 0.10, 0.142 0.19, 0.25 0.33, 0.365 0.5	0.2, 0.25 0.3, 0.35	2.3, 2.8 4.3, 6.4	1,520	100–2,000	0.7–50	1.0–7.0

Herringe considered that speed of N_{cs} at which "no solids remained stationary on the bottom for more than 2 s." With Zwietering's criterion, almost all the particles are suspended at N_{cs} and some particles are permitted to stay on the bottom for less than 2 s. With Herringe's criterion, the solids are permitted to stay on the bottom (not all in suspension) but they should be in motion, never stationary for a time period exceeding 2 s.

Raghav Rao et al. (1988) have studied the effect of blade width. They found that the value of N_{cs} varies as the square root of blade width when W/D is less than 0.35. Using this relationship, our values appear to be 10 to 15% higher than those of Herringe (1979). This difference is small and can be considered to be due to the difference in the suspension criterion.

5. Raghav Rao et al. (1988) found the exponent on D to be -1.16 for constant, T. Zwietering (1958) has reported a value of -1.67, using a propeller. Chap-

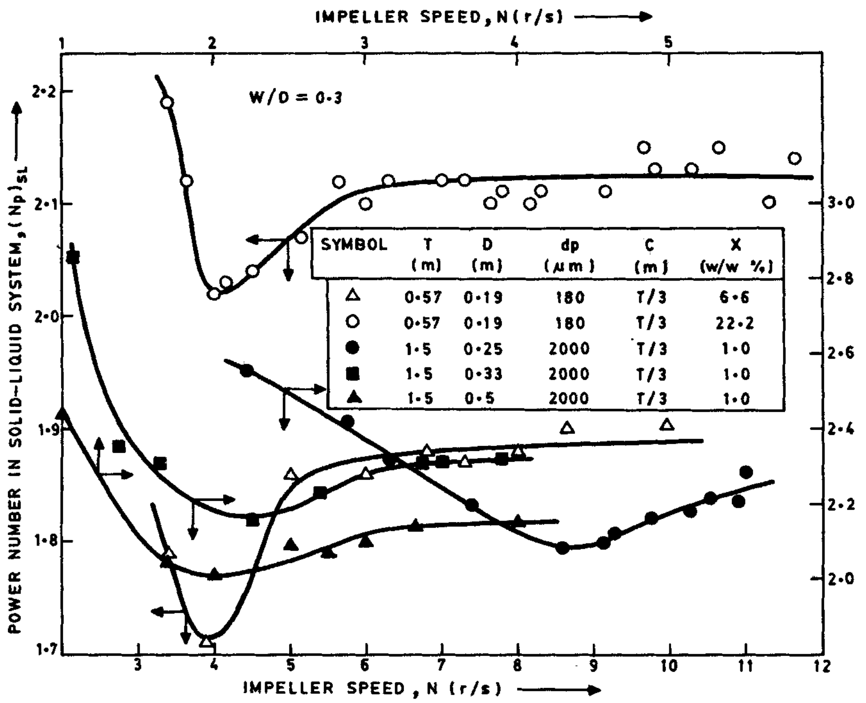

Figure 1. Variation of power number with impeller speed at constant loading.

man et al. (1983) observed a value of −1.5, using four-bladed pitched downflow turbine with W/D ratio of 0.3. This difference is probably because of the difference in flow pattern. We also found that the value of the exponent decreases with an increase in D. This is because the smaller diameter impellers generate relatively more axial flow in the same tank. Further, for same D/T, the increase in vessel diameter results in more axial flow. When we correlate the N_{cs} data for only 1.5 m I.D. vessels ($1/6 < D/T < 1/2$, $W/D = 0.3$, $C = T/3$), the following equation was obtained:

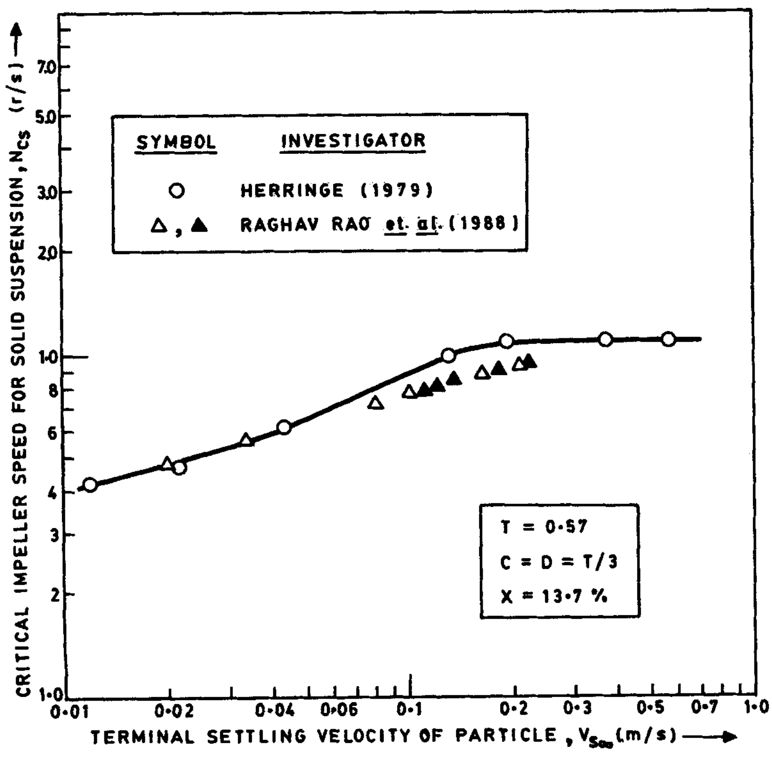

Figure 2. Effect of settling velocity on critical impeller speed for solid suspension.

$$N_{cs} = C_2 \frac{V_{S\infty}^{0.28} X^{0.1}}{D^{0.85}} \left(\frac{T}{D}\right)^{0.8} \tag{2}$$

where the average value of C_2 was found to be 1.15 ($S.D.$ = 5%)

We are of the opinion that "a certain proportion of mixed flow" is more suited for solid suspension and that there is a possibility of optimizing the pitched blade design. It may be pointed out that Eq. 2 boils down to Eq. 1 when T/D is 3.

We did not cite the work of Gates et al. (1976) because his correlation includes two impeller designs and also some data using multiple impellers.

Chapman et al. (1983) found the exponent to be −1.65 for a propeller and −1.5 for four-bladed pitched turbine. Thus, the value of exponent was found to decrease with some addition of radial flow. Therefore, these authors have rightly recommended mixed flow turbine. We feel that the exponent decreases further when six-bladed pitched turbine is used in place of four blades.

6. The discrepancy pointed out by Bowen (1989) regarding the scaleup criterion gets resolved if we analyse the experimental results more closely. Raghav Rao et al. (1988) have found the exponent on T as −0.85 (to be more precise, the exponent is obtained on D at constant D/T). This value remained practically the same in the particle size range of 100–2,200 μm. Further, Raghav Rao et al. (1988) obtained the exponent on T by covering vessel diameters over a wide range (0.3, 0.57, 1.0, and 1.5 m). Chapman et al. (1983) also covered three vessel sizes (0.29, 0.56, and 0.91 m) to arrive at the exponent, −0.76. Zwietering (1958) used six vessels while arriving at the exponent, −0.85. Gates et al. (1976) have given a value of −0.75 again by covering many vessel sizes.

However, this is not the case for the studies of Herringe (1979) and Kneule and Weinspach (1967). It appears that the experiments were performed on a single scale. It is difficult to develope scale-up criterion without changing scale. Herringe (1979) has changed impeller size in the same vessel for getting the scale-up exponent, −0.4. When the impellers are 50 to 100 mm size, the design details become very important. Further, the estimated value of -0.4 (in the case of Herringe, 1979) for large particles is based on very few experimental results.

Chudacek (1986) has reported that the scale-up criterion depends upon the suspension criterion. For three suspensions criteria, he has given the exponent on T close to -0.75, whereas for one criterion he has given a value of -0.58.

Buurman et al. (1985) have found the exponent, -0.67, even for small particles. The design of bottom and baffles was different from all the above investigators and the scale-up criterion cannot be compared directly.

Notation

C = impeller clearance from tank bottom, m
C_1 = proportionality constant, Eq. 1
C_2 = proportionality constant, Eq. 2
D = impeller diameter, m
N_{CS} = critical impeller speed for solid suspension, r/s
T = tank diameter, m
$V_{s\infty}$ = particle terminal settling velocity, m/s
W = blade width, m
X = solid loading, wt. %
ρ_L = density of liquid, kg/m^3
ρ_s = density of solid, kg/m^3
$\Delta\rho = \rho_s - \rho_L$

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V. B. Rewatkar, K. S. M. S. Raghav Rao, and J. B. Joshi
Department of Chemical Technology
University of Bombay
Matunga, Bombay 400 019, India

***Editor's Note:** Dr. Bowen has made the following (edited) observations at the proof stage regarding the authors' response.*

Item 3 of Your Reply. The settling velocity data of Raghava Rao et al. (1988, Table 3) do not agree with those of Herringe in his Table 1 (which correspond to the theoretical values computed from Stokes' and Newton's laws). . . . It appears that Herringe's settling velocity data were used for his data in the authors' Figure 2, while Raghava Rao et al.'s settling velocity data were used for their data, so the development of the two sets of N_{CS} data is not comparable. Furthermore, the development of $V_S^{0.28}$ appears to be based on Raghav Rao's settling velocities. . . . Use of the theoretically calculated settling velocity data should decrease the exponent of $V_S^{0.10}$.

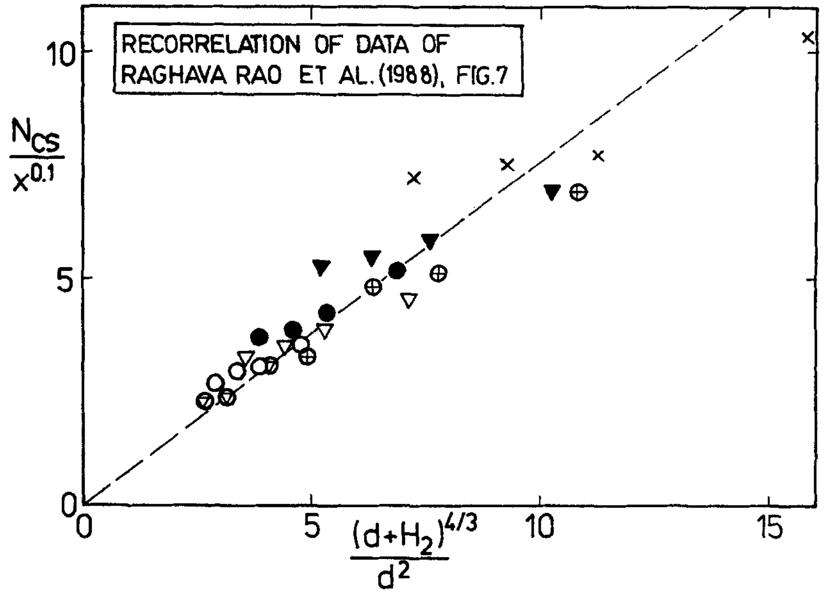
Item 4. The authors have misunderstood the word "stationary". . . . Herringe's summary (p. 199) states that "The critical speed for complete suspension of solids was recorded." "Complete" means no solids on the bottom for more than 2 s. Therefore, "stationary" means in the vertical direction, not the horizontal plane of the bottom (c.f., Chudacek, 1986, p. 396). . . . : "Complete off-bottom suspension, defined as the impeller speed necessary to insure that no solids remain stationary on the tank bottom for longer than 2 s."

Item 6. Herringe clearly states (p. 202) that multiple vessels were used. One impeller was run in each vessel, as D/T was 1/3 for all vessels.

To the Editor:

In the paper entitled, "Critical Impeller Speed for Solid Suspension in Mechanically Agitated Contactors," Raghava Rao, Rewatkar and Joshi [**34** (8), 1332, Aug., 1988] presented data on critical rotation speed, N_{CS}, for suspension of solids in agitated vessels. In contrast to previous literature data whose main focus had been on the effects of particle and liquid properties, here, effects of the vessel and impeller geometry were studied more thoroughly. Though a considerable set of data was collected, no general correlation was suggested and only qualitative conclusions were made.

Recently we have developed (Wichterle, 1988) a theoretical model for solid suspension. According to this model, the effects of the impeller and tank arrangement can be expressed by a single quantity: critical shear rate at the tank bottom. For axial impellers pumping downward, which is the favorite system for the solid suspension, the shear rates, γ, at the



walls, were measured and the correlation

$$\gamma_{max} = 1.6N\left(\frac{d}{d + H_2}\right)^2\left(\frac{Nd^2\rho}{\mu}\right)^{1/2}$$

was suggested for various tank diameters, D, impeller diameters, d, and impeller-to-bottom clearance, H_2 (Wichterle et al., 1988). If the critical share rate for suspension is considered to be proportional to γ_{max}, then for the critical impeller speed, the proportionality

$$N_{CS} \sim \frac{(d + H_2)^{4/3}}{d^2}$$

should hold. The data of Raghava Rao, Rewatkar, and Joshi (1988), for D = 0.57–1 m, d/D = 0.25–0.6, H_2/D = 0.16–0.5, and for two different concentrations of the solid phase, X, rearranged in Figure 1, indicate that such a proportionality could be accepted. This is further support for using the concept of critical shear rate in considerations related to suspension of solid particles.

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Kamil Wichterle
Institute of Chemical Process Fundamentals
Czechoslovak Academy of Sciences
16502 Prague, Czechoslovakia

Reply:

We commend the outstanding efforts made by Wichterle (1988b) and Wichterle et al. (1988a) in developing rational correlation for solid suspension. They have, perhaps, presented initial results of an exercise of continuing nature. The correlation in the present form needs modifications to include the following:

i. The correlation based on theoretical consideration usually has a better predictive ability for scale-up. Therefore, the effect of vessel size on N_{cs} is very important and needs to be included in the correlation given by:

$$N_{cs} \sim \frac{(d + H_2)^{4/3}}{d^2}$$

Further, the dependence of N_{cs} on vessel size is decided by the impeller design.

ii. The effect of impeller diameter on N_{cs} is again dependent on the impeller design. It is difficult to have a generalized dependence.

iii. The mechanism of solid suspension as viewed in the mathematical model, needs to be consistent with experimental observations. Wichterle (1988b) correctly suggests that the point of minimum shear rate should be considered, where the particles will be suspended last. He has given plots of shear rate with respect to radial location for downflow and disc turbines. The points of minimum shear rate shown on this graph do not generally agree with the experimentally observed points of last suspension.

iv. The mathematical model of Wichterle (1988b) assumes laminar boundary layer of thickness equal to at least particle size. Therefore, the variation of N_{cs} and d_p and μ_L, is strongly governed by the assumption of laminar boundary layer. And thus, the exponents on d_p and μ_L as listed in Table 1 of Wichterle, are markedly different from those observed experimentally. For instance, Gray and Oldshue (1986) have summarized the values of exponents in their excellent review (Table VIII).

v. Wichterle (1988b) has given the following equation for minimum shear rate:

$$\gamma_{min} = A_{min}\,N\left(\frac{Nd^2\rho_L}{\mu_L}\right)^{0.5}\left(\frac{d}{D}\right)^2$$

The value of A_{min} for disc turbine was found to be 2.5, and for pitched bladed turbine downflow (*PTD*), A_{min} was 3.5. This indicates that the values of N_{cs} for disc turbine and for *PTD* will differ by a constant factor when variations are made in the impeller size, particle size, and solid loading. This is not usually observed in practice (Raghav Rao et al., 1988).

vi. Wicherle (1988b) has made an excellent contribution to the subject by developing a model for solid suspension on the basis of flow structure in the vicinity of bottom. There are some models which explain the solid suspension on the basis of flow in the main body of the vessel, e.g., Subbarao and Taneja (1976), Voit and Mersmann (1986), Molerus and Latzel (1987). Chudacek (1986) has pointed out the limitations of these models which base the solid suspension mechanisms on either of the flows. These limitations get substantially reduced when the suspension mechanism is based on flow structure in the entire vessel. Recently, Ranade and Joshi (1989a, b, c, d) have measured the flow pattern in agitated vessels using Laser-Doppler Anemometer. On the basis of these measurements, Rewatkar et al. (1989) have developed a mathematical model for solid suspension.

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V. B. Rewatkar and J. B Joshi
Department of Chemical Technology
University of Bombay
Matunga, Bombay 400 019, India

Errata

In the paper entitled "Multicomponent Reactive Dispersion in Tubes: Collocation vs. Radial Averaging" by J. Chih Wang and Warren E. Stewart (**35**, March, 1989, p. 490), the following corrections are made:

The last term of Eq. 38 on p. 494 should end with $(\lambda_i - \lambda_k)$, not $(\lambda_i - \lambda_j)$.

Under Eq. 44 on p. 495, ξ^{2n-4} should be changed to ξ^{2n-2}.

"Model matrix" on the second line of the righthand column of p. 495 should read "modal matrix."

The following entry should be added to the "Literature Cited" Section on pp. 497–8: Wang, J. C., and W. E. Stewart, "New Descriptions of Dispersion in Flow Through Tubes: Convolution and Collocation Methods," *AIChE J.*, **29**, 493 (May, 1983).

In Eq. B3 on p. 498, $\xi = \mathbf{0}$ should read $\xi = 0$.

The second line of the righthand column on p. 499 should read, "on the interval (0, 1). So $\langle \mathbf{Tu}, \mathbf{u} \rangle$ is nonpositive." The seventh line of this column should start with $\mathbf{K}'''$, not $\mathbf{K}''$. In the ninth line, w should read ω.